1. Find the following limit. (If the limit does not exist or has an infinite limit, you should point it out) (12%)

   (a) $\lim_{x\to 3} \frac{x^2+14x-51}{x^3-5x^2+4x+6}$

   (b) $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x}$

   (c) $\lim_{x\to 0} x\left(\cos 2x + \cos\frac{1}{x}\right)$

   (d) $\lim_{x\to 2} \frac{2x+6}{x-2}$

2. Determine all values of the constant a such that the following function is continuous at $x = 0$. (8%)

$$f(x) = \begin{cases} a^2 - 2, & x < 0 \\ \dfrac{ax}{\tan x}, & x \geq 0 \end{cases}$$

3. Proof that there is only one intersect point between $f(x) = 5x^3 + 2x^2 + 4x + 1$ and $g(x) = 2x^2 + \cos x$. (Hint: use the mean value theorem) (10%)

4. Remember that you can solve the derivative using the definition or the differentiation rule for the following question. (12%)

   (a) Given $f(x) = \frac{x}{(x+1)(x+2)\ldots(x+2021)}$, what is the value of $f'(0)$?

   (a) Find the derivative of $f(x) = \frac{x^3+5x+3}{x^2-1}$

   (b) Find the derivative of $f(x) = \sin(\sqrt{cot(5\pi x)})$

   (c) Find the following limit. $\lim_{x\to 0} \frac{sin(\sqrt{3+x})-sin(\sqrt{3})}{x}$

5. Given $x^2y^3 - 5xy^2 - 4y = 4$, find the tangent line at (3,2). (10%)

6. Let $f(x) = \frac{2x^2}{x^2-1}$ (10%)

   (a) Find the critical numbers and the possible points of inflection of $f(x)$

   (b) Find the open intervals on which f is increasing or decreasing

   (c) Find the open intervals on which f is concave upward or concave downward

   (d) Find all the asymptotes (Both vertical and horizontal)

   (e) Sketch the graph of $f(x)$

7. Prove that $|tanx - tany| \geq |x - y|$ for all $x, y \in (\frac{-\pi}{2}, \frac{\pi}{2})$. (Hint: use the mean value theorem) (10%)

8. (12%)

   (a) $\int \frac{1+x+x^2}{\sqrt{x}} dx$

   (b) $\int 6\text{t-}\csc^2\text{t}\, dt$

   (c) Evaluate $\lim_{n\to\infty} \frac{1}{\sqrt{n}}(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}})$ (Hint: use Riemann sum and the definition of the definite integral)

   (d) $\int_{-\pi}^{\pi} \frac{x^3 cosx}{1+x^4} dx$

9. Find $\frac{d}{dx}\int_{2x}^{x^2} cos\sqrt{t}\, dt$ when $x > 0$. (Hint: Let Let $F(x) = \int_1^x cos\sqrt{t} dt$ and use the fundamental theorem of calculus) (8%)

10. Evaluate $\int_1^2 2x^2\sqrt{x^3+1} dx$. (8%)