1213 HW-A

1. Evaluate the limit, using L'Hopital's Rule if necessary.

$$\lim_{x\to\infty}(1+x)^{\frac{1}{x}}$$

2. Find the derivative of the function.

$$f(x) = x^2 \arctan 5x$$

3. Find or evaluate the integral

   (a) $\displaystyle\int \frac{1}{\sqrt{-2x^2+8x+4}}\,dx$

   (b) $\displaystyle\int_0^1 \frac{1}{2\sqrt{3-x}\sqrt{x+1}}\,dx$

sol:

1.

$$\begin{aligned}
\text{Let } y &= \lim_{x\to\infty}(1+x)^{\frac{1}{x}} \\
\ln y &= \lim_{x\to\infty}\frac{\ln(1+x)}{x} = \lim_{x\to\infty}\left(\frac{1/(1+x)}{1}\right) = 0 \\
\text{S0, } \ln y &= 0 \Rightarrow y = e^0 = 1 \\
\text{Therefore, } &\lim_{x\to\infty}(1+x)^{\frac{1}{x}} = 1
\end{aligned}$$

2.

$$\begin{aligned}
f(x) &= x^2 \arctan 5x \\
f'(x) &= 2x\arctan 5x + x^2\frac{5}{1+(5x)^2} \\
&= 2x\arctan 5x + \frac{5x^2}{1+25x^2}
\end{aligned}$$

3. (a)

$$\begin{aligned}
\int \frac{1}{\sqrt{-2x^2+8x+4}}\,dx &= \int \frac{1}{\sqrt{12-2(x^2-4x+4)}}\,dx \\
&= \frac{1}{\sqrt{2}}\int\frac{1}{\sqrt{6-(x-2)^2}}\,dx \\
&= \frac{1}{\sqrt{2}}\arcsin\left(\frac{x-2}{\sqrt{6}}\right)+C \\
&= \frac{\sqrt{2}}{2}\arcsin\left[\frac{\sqrt{6}}{6}(x-2)\right]+C
\end{aligned}$$

   (b)

$$\text{Let } u=\sqrt{x+1}, u^2 = x+1, 2u\ du = dx, \sqrt{3-x}=\sqrt{4-u^2}$$

$$\begin{aligned}
\int_1^{\sqrt{2}}\frac{2u}{2\sqrt{4-u^2}u}\,dx &= \int_1^{\sqrt{2}}\frac{1}{\sqrt{4-u^2}}\,du \\
&= \arcsin\left(\frac{u}{2}\right)|_1^{\sqrt{2}} \\
&= \arcsin\left(\frac{\sqrt{2}}{2}\right) - \arcsin\left(\frac{1}{2}\right) \\
&= \frac{\pi}{4}-\frac{\pi}{6} = \frac{\pi}{12}
\end{aligned}$$