1213 HW-B

1. Evaluate the limit, using L'Hopital's Rule if necessary.

$$\lim_{x\to 4^+} [3(x-4)]^{x-4}$$

2. Find the derivative of the function.

$$f(x) = \operatorname{arccot} \sqrt{x}$$

3. Find or evaluate the integral

   (a) $\displaystyle\int \frac{1}{3x^2 - 6x + 12}\, dx$

   (b) $\displaystyle\int_1^3 \frac{1}{\sqrt{x}(1+x)}\, dx$

sol:

1.

$$\begin{aligned}
\text{Let } y &= \lim_{x\to 4^+} [3(x-4)]^{x-4} \\
\ln y &= \lim_{x\to 4^+} (x-4)\ln[3(x-4)] \\
&= \lim_{x\to 4^+} \frac{\ln[3(x-4)]}{1/(x-4)} \\
&= \lim_{x\to 4^+} \frac{1/(x-4)}{-1/(x-4)^2} \\
&= \lim_{x\to 4^+} [-(x-4)] = 0 \\
\text{S0, } & \lim_{x\to 4^+} [3(x-4)]^{x-4} = 1
\end{aligned}$$

2.

$$\begin{aligned}
f(x) &= \operatorname{arccot} \sqrt{x} \\
f'(x) &= \frac{-1/2\sqrt{x}}{1+x} \\
&= -\frac{1}{2\sqrt{x}(1+x)}
\end{aligned}$$

3. (a)

$$\begin{aligned}
\int \frac{1}{3x^2-6x+12}\, dx &= \frac{1}{3}\int \frac{1}{x^2-2x+4}\, dx \\
&= \frac{1}{3}\int \frac{1}{(x^2-2x+1)+3}\, dx \\
&= \frac{1}{3}\int \frac{1}{(x-1)^2+3}\, dx \\
&= \frac{1}{3}\left(\frac{1}{\sqrt{3}}\right) \arctan\frac{x-1}{\sqrt{3}} + C \\
&= \frac{\sqrt{3}}{9} \arctan\left[\frac{\sqrt{3}}{3}(x-1)\right] + C
\end{aligned}$$

   (b)

$$\begin{aligned}
&\text{Let } u = \sqrt{x}, u^2 = x, 2u\ du = dx, 1+x = 1+u^2 \\
\int_1^{\sqrt{3}} \frac{2u}{u(1+u^2)}\, du &= \int_1^{\sqrt{3}} \frac{2}{1+u^2}\, du \\
&= [2\arctan(u)]|_1^{\sqrt{3}} \\
&= 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\pi}{6}
\end{aligned}$$