1213 HW-C

1. Evaluate the limit, using L'Hopital's Rule if necessary.

$$\lim_{x\to 2^+}\left(\frac{1}{x^2-4}-\frac{\sqrt{x-1}}{x^2-4}\right)$$

2. Find the derivative of the function.

$$f(x) = \text{arcsec } 2x$$

3. Find or evaluate the integral

   (a) $\displaystyle\int \frac{2}{\sqrt{-x^2+4x}}\,dx$

   (b) $\displaystyle\int_3^4 \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx$

sol:

1.

$$\begin{aligned}\lim_{x\to 2^+}\left(\frac{1}{x^2-4}-\frac{\sqrt{x-1}}{x^2-4}\right) &= \lim_{x\to 2^+}\frac{1-\sqrt{x-1}}{x^2-4}\\ &= \lim_{x\to 2^+}\frac{-1/(2\sqrt{x-1})}{2x}\\ &= \lim_{x\to 2^+}\frac{-1}{4x\sqrt{x-1}}\\ &= -\frac{1}{8}\end{aligned}$$

2.

$$\begin{aligned}f(x) &= \text{arcsec } 2x\\ f'(x) &= \frac{2}{|2x|\sqrt{4x^2-1}} = \frac{1}{|x|\sqrt{4x^2-1}}\end{aligned}$$

3. (a)

$$\begin{aligned}\int \frac{2}{\sqrt{-x^2+4x}}\,dx &= \int \frac{2}{\sqrt{4-(x^2-4x+4)}}\,dx\\ &= \int \frac{2}{\sqrt{4-(x-2)^2}}\,dx = 2\arcsin\left(\frac{x-2}{2}\right)+C\end{aligned}$$

   (b)

$$\begin{aligned}\int_3^4 \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx &= \int_3^4 \frac{1}{(x-1)\sqrt{x^2-2x+1-1}}\,dx\\ &= \int_3^4 \frac{1}{(x-1)\sqrt{(x-1)^2-1}}\,dx\\ &= [\text{arcsec}|x-1|]|_3^4\\ &= \text{arcsec}3-\text{arcsec}2\end{aligned}$$