1206 HW-C

1. Use the function f and the given real number a to find $(f^{-1})'(a)$.

$$f(x) = \sqrt{x-4}\ ,\ a = 2$$

2. Find the derivative of the function.

   (a) $y = \dfrac{e^{2x}}{e^{2x}+1}$

   (b) $y = \log_3(x^2 - 3x)$

3. Find the integral.

$$\int_{-2}^{1} x^2 e^{\frac{x^3}{2}}\ dx$$

sol:

1.

$$\begin{aligned} f(x) &= \sqrt{x-4}\ ,\ a = 2 \\ f'(x) &= \frac{1}{2\sqrt{x-4}} > 0 \quad \text{on } (4,\infty) \\ f(8) &= 2 \Rightarrow f^{-1}(2) = 8 \\ f'(8) &= \frac{1}{2\sqrt{8-4}} = \frac{1}{4} \\ (f^{-1})'(2) &= \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(8)} = \frac{1}{\frac{1}{4}} = 4 \end{aligned}$$

2. (a)

$$\begin{aligned} y &= \frac{e^{2x}}{e^{2x}+1} \\ y' &= \frac{(e^{2x}+1)2e^{2x} - e^{2x}(2e^{2x})}{(e^{2x}+1)^2} = \frac{2e^{2x}}{(e^{2x}+1)^2} \end{aligned}$$

   (b)

$$\begin{aligned} y &= \log_3(x^2-3x) \\ y' &= \frac{1}{(x^2-3x)\ln 3}(2x-3) \\ &= \frac{2x-3}{x(x-3)\ln 3} \end{aligned}$$

3.

$$\begin{aligned} \int_{-2}^{1} x^2 e^{\frac{x^3}{2}}\ dx &= \frac{2}{3}\int_{-2}^{1} e^{\frac{x^3}{2}}\left(\frac{2}{3}x^2\right)\ dx \\ &= \frac{2}{3}\left[e^{\frac{x^3}{2}}\right]_{-2}^{1} \\ &= \frac{2}{3}\left(\sqrt{e} - e^{-4}\right) \end{aligned}$$