1206 HW-A

1. Use the function f and the given real number a to find $(f^{-1})'(a)$.

$$f(x) = \frac{x+6}{x-2} \ , \ x > 2 \ , \ a = 3$$

2. Find the derivative of the function.

   (a) $y = \ln \dfrac{1+e^x}{1-e^x}$

   (b) $y = \log_{10} \dfrac{x^2-1}{x}$

3. Find the integral.

$$\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} \, dx$$

sol:

1.

$$\begin{aligned}
f(x) &= \frac{x+6}{x-2} \ , \ x > 2 \ , \ a = 3 \\
f'(x) &= \frac{(x-2)(1) - (x+6)(1)}{(x-2)^2} \\
&= \frac{-8}{(x-2)^2} < 0 \quad \text{on } (2, \infty) \\
f(6) &= 3 \Rightarrow f^{-1}(3) = 6 \\
(f^{-1})'(3) &= \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(6)} = \frac{1}{\frac{-8}{(6-2)^2}} = -2
\end{aligned}$$

2. (a)

$$\begin{aligned}
y &= \ln \frac{1+e^x}{1-e^x} = \ln(1+e^x) - \ln(1-e^x) \\
\frac{dy}{dx} &= \frac{e^x}{1+e^x} + \frac{e^x}{1-e^x} = \frac{2e^x}{1-e^{2x}}
\end{aligned}$$

   (b)

$$\begin{aligned}
y &= \log_{10} \frac{x^2-1}{x} = \log_{10}(x^2-1) - \log_{10} x \\
\frac{dy}{dx} &= \frac{2x}{(x^2-1)\ln 10} - \frac{1}{x \ln 10} \\
&= \frac{1}{\ln 10} \left[\frac{2x}{x^2-1} - \frac{1}{x} \right] \\
&= \frac{1}{\ln 10} \left[\frac{x^2+1}{x(x^2-1)} \right]
\end{aligned}$$

3.

$$\begin{aligned}
&\text{Let } u = e^x + e^{-x}, du = (e^x - e^{-x})dx \\
\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} \, dx &= 2 \int (e^x + e^{-x})^{-2} (e^x - e^{-x}) \, dx \\
&= \frac{-2}{e^x + e^{-x}} + C
\end{aligned}$$