1206 HW-B

1. Use the function f and the given real number a to find $(f^{-1})'(a)$.

$$f(x) = \frac{x^5 + 2x^3}{27} \ , \ a = -11$$

2. Find the derivative of the function.

   (a) $y = \dfrac{2}{e^x + e^{-x}}$

   (b) $y = \log_2 \sqrt[3]{2x+1}$

3. Find the integral.

$$\int \frac{e^{x^{-2}}}{x^3} \, dx$$

sol:

1.

$$\begin{aligned} f(x) &= \frac{x^5 + 2x^3}{27} \ , \ a = -11 \\ f'(x) &= \frac{5x^4 + 6x^2}{27} \\ f(-3) &= \frac{-243 - 54}{27} = -11 \Rightarrow f^{-1}(-11) = -3 \\ (f^{-1})'(-11) &= \frac{1}{f'(f^{-1}(-11))} = \frac{1}{f'(-3)} = \frac{1}{\frac{(5(-3)^4 + 6(-3)^2)}{27}} = \frac{1}{17} \end{aligned}$$

2. (a)

$$\begin{aligned} y &= \frac{2}{e^x + e^{-x}} = 2(e^x + e^{-x})^{-1} \\ \frac{dy}{dx} &= -2(e^x + e^{-x})^{-2}(e^x - e^{-x}) = \frac{-2(e^x - e^{-x})}{(e^x + e^{-x})^2} \end{aligned}$$

(b)

$$\begin{aligned} y &= \log_2 \sqrt[3]{2x+1} = \frac{1}{3} \log_2(2x+1) \\ y' &= \frac{1}{3} \frac{1}{(2x+1)\ln 2}(2) = \frac{2}{3(2x+1)\ln 2} \end{aligned}$$

3.

$$\text{Let } u = \frac{1}{x^2}, du = \frac{-2}{x^3} \ dx$$

$$\int \frac{e^{x^{-2}}}{x^3} \ dx = -\frac{1}{2} \int e^{x^{-2}} \left(\frac{-2}{x^3} \right) \ dx = -\frac{1}{2} e^{x^{-2}} + C$$