1129 HW-A

1. Find the derivative of the function
$$y = \ln\left(x\sqrt{x^2-1}\right)$$

2. Use implicit differentiation to find $\dfrac{dy}{dx}$
$$4x^3 + \ln y^2 + 2y = 2x$$

3. Find the integral
   (a) $\displaystyle\int \frac{1}{x(\ln x^2)^3}\,dx$
   (b) $\displaystyle\int \frac{\sec x \tan x}{\sec x - 1}\,dx$

sol:

1.
$$y = \ln\left(x\sqrt{x^2-1}\right) = \ln x + \frac{1}{2}\ln\left(x^2-1\right)$$
$$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\left(\frac{2x}{x^2-1}\right) = \frac{2x^2-1}{x(x^2-1)}$$

2.
$$4x^3 + \ln y^2 + 2y = 2x$$
$$12x^2 + \frac{2}{y}y' + 2y' = 2$$
$$\left(\frac{2}{y} + 2\right)y' = 2 - 12x^2$$
$$y' = \frac{2-12x^2}{2/y+2}$$
$$y' = \frac{y - 6yx^2}{1+y} = \frac{y(1-6x^2)}{1+y}$$

3. (a)
$$u = \ln x^2, du = \frac{2x}{x^2}dx = \frac{2}{x}$$
$$\int \frac{1}{x(\ln x^2)^3}\,dx = \frac{1}{2}\int (\ln x^2)^{-3}\frac{2}{x}dx$$
$$= \frac{1}{2}\frac{(\ln x^2)^{-2}}{(-2)} + C$$
$$= \frac{-1}{4(\ln x^2)^2} + C$$

(b)
$$u = \sec x - 1, du = \sec x \tan x dx$$
$$\int \frac{\sec x \tan x}{\sec x - 1}\,dx = \ln|\sec x - 1| + C$$