1129 HW-C

1. Find the derivative of the function
$$y = \ln \frac{2x}{x+3}$$

2. Use implicit differentiation to find $\frac{dy}{dx}$
$$x^2 - 3\ln y + y^2 = 10$$

3. Find the integral

(a) $\int \frac{x(x-2)}{(x-1)^3} \, dx$

(b) $\int \frac{\cos x}{1+\sin x} \, dx$

sol:

1.
$$y = \ln \frac{2x}{x+3} = \ln 2x - \ln x + 3$$
$$y' = \frac{1}{x} - \frac{1}{x3} = \frac{3}{x(x+3)}$$

2.
$$x^2 - 3\ln y + y^2 = 10$$
$$2x - \frac{3}{y}\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$
$$2x = \frac{dy}{dx}\left(\frac{3}{y} - 2y\right)$$
$$\frac{dy}{dx} = \frac{2x}{(3/y) - 2y} = \frac{2xy}{3 - 2y^2}$$

3. (a)
$$\int \frac{x(x-2)}{(x-1)^3} \, dx = \int \frac{x^2 - 2x + 1 - 1}{(x-1)^3} \, dx$$
$$= \int \frac{(x-1)^2}{(x-1)^3} \, dx - \int \frac{1}{(x-1)^3} \, dx$$
$$= \int \frac{1}{x-1} \, dx - \int \frac{1}{(x-1)^3} \, dx$$
$$= \ln|x-1| + \frac{1}{2(x-1)^2} + C$$

(b)
$$u = 1 + \sin x, \, du = \cos x \, dx$$
$$\int \frac{\cos x}{1+\sin x} \, dx = \ln|1+\sin x| + C$$