1129 HW-B

1. Find the derivative of the function
$$y = \ln \frac{x}{x^2+1}$$

2. Use implicit differentiation to find $\dfrac{dy}{dx}$
$$\ln xy + 5x = 30$$

3. Find the integral

   (a) $\displaystyle\int \frac{1}{\sqrt[3]{x^2}(1+\sqrt[3]{x})}\ dx$

   (b) $\displaystyle\int \left(2 - \tan\frac{x}{4}\right)\ dx$

sol:

1.
$$y = \ln \frac{x}{x^2+1} = \ln x - \ln x^2 + 1$$
$$y' = \frac{1}{x} - \frac{2x}{x^2+1} = \frac{1-x^2}{x(x^2+1)}$$

2.
$$\begin{aligned}
\ln xy + 5x &= 30 \\
\ln x + \ln y + 5x &= 30 \\
\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} + 5 &= 0 \\
\frac{1}{y}\frac{dy}{dx} &= -\frac{1}{x} - 5 \\
\frac{dy}{dx} &= -\frac{y}{x} - 5y = -\left(\frac{y+5xy}{x}\right)
\end{aligned}$$

3. (a)
$$u = 1+\sqrt[3]{x}, du = \frac{1}{3\sqrt[3]{x^2}}dx$$
$$\begin{aligned}
\int \frac{1}{\sqrt[3]{x^2}(1+\sqrt[3]{x})}\ dx &= 3\int \frac{1}{1+\sqrt[3]{x}}\left(\frac{1}{3\sqrt[3]{x^2}}\right)dx \\
&= 3\ln|1+\sqrt[3]{x}| + C
\end{aligned}$$

   (b)
$$\begin{aligned}
\int \left(2 - \tan\frac{x}{4}\right)\ dx &= \int 2dx - 4\int \tan\frac{x}{4}\left(\frac{1}{4}\right)dx \\
&= 2x + 4\ln|\cos\frac{x}{4}| + C
\end{aligned}$$