1101 HW-B

1. Find the indefinite integral

   (a) $\int (\sqrt[4]{x^3} + 1)\ dx$

   (b) $\int (\sec x)(\tan x - \sec x)\ dx$

2. Use the limit process to find the area of the region bounded by the graph of the function and the x-axis over the given interval.

$$y = 5x^2 + 1,\ [0, 2]$$

sol:

1. (a)

$$\int (\sqrt[4]{x^3} + 1)\ dx = \int \left(x^{\frac{3}{4}} + 1\right)\ dx = \frac{4}{7}x^{\frac{7}{4}} + x + C$$

   (b)

$$\int (\sec x)(\tan x - \sec x)\ dx = \int \left(\sec x \tan x - \sec^2 x\right)\ dx = \sec x - \tan x + C$$

2.

$$\begin{aligned}
y = 5x^2 + 1,\ [0,2].& \left(\text{Note:}\Delta x = \frac{2-0}{n} = \frac{2}{n}\right) \\
S(n) &= \sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right) \\
&= \sum_{i=1}^{n}\left[5\left(\frac{2i}{n}\right)^2 + 1\right]\left(\frac{2}{n}\right) \\
&= \frac{40}{n^3}\sum_{i=1}^{n} i^2 + \frac{2}{n}\sum_{i=1}^{n} 1 \\
&= \frac{40}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{2}{n}(n) \\
&= \frac{20}{3n^2}(n+1)(2n+1) + 2 \\
\text{Area} &= \lim_{n\to\infty} S(n) = \frac{40}{3} + 2 = \frac{46}{3}
\end{aligned}$$