1101 HW-A

1. Find the indefinite integral

   (a) $\displaystyle\int \frac{x^4 - 3x^2 + 5}{x^4}\, dx$

   (b) $\displaystyle\int (4x - \csc^2 x)\, dx$

2. Use the limit process to find the area of the region bounded by the graph of the function and the x-axis over the given interval.

$$y = 3x - 2,\ [2, 5]$$

sol:

1. (a)

$$\begin{aligned}\int \frac{x^4 - 3x^2 + 5}{x^4}\, dx &= \int \left(1 - 3x^{-2} + 5x^{-4}\right)\, dx \\ &= x - \frac{3x^{-1}}{-1} + \frac{5x^{-3}}{-3} + C \\ &= x + \frac{3}{x} - \frac{5}{3x^3} + C\end{aligned}$$

   (b)

$$\int (4x - \csc^2 x)\, dx = 2x^2 + \cot x + C$$

2.

$$\begin{aligned} y = 3x - 2,\ [2,5].\ &\left(\text{Note:}\Delta x = \frac{5-2}{n} = \frac{3}{n}\right) \\ S(n) &= \sum_{i=1}^{n} f\left(2 + \frac{3i}{n}\right)\left(\frac{3}{n}\right) \\ &= \sum_{i=1}^{n}\left[3\left(2 + \frac{3i}{n}\right) - 2\right]\left(\frac{3}{n}\right) \\ &= 18 + 3\left(\frac{3}{n}\right)^2 \sum_{i=1}^{n} i - 6 \\ &= 12 + \frac{27}{n^2}\left(\frac{(n+1)n}{2}\right) \\ &= 12 + \frac{27}{2}\left(1 + \frac{1}{n}\right) \\ \text{Area} &= \lim_{n\to\infty} S(n) = 12 + \frac{27}{2} = \frac{51}{2}\end{aligned}$$