1101 HW-C

1. Find the indefinite integral

   (a) $\int (4x^2+3)^2 \, dx$

   (b) $\int (x^2+\sec^2 x) \, dx$

2. Use the limit process to find the area of the region bounded by the graph of the function and the x-axis over the given interval.

$$y = 4 - x^2, \ [-2, 2]$$

sol:

1. (a)

$$\int (4x^2+3)^2 \, dx = \int (16x^4+24x^2+9) \, dx = \frac{16x^5}{5} + 8x^3 + 9x + C$$

   (b)

$$\int (x^2+\sec^2 x) \, dx = \frac{1}{3}x^3 + \tan x + C$$

2.

$$\begin{aligned} y = 4 - x^2, \, [-2,2]. \left(\text{Note:}\Delta x = \frac{2}{n}\right) \\ S(n) &= \sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right) \\ &= \sum_{i=1}^{n} \left[4 - \left(\frac{2i}{n}\right)^2\right]\left(\frac{2}{n}\right) \\ &= 8 - \frac{8}{n^3}\sum_{i=1}^{n} i^2 \\ &= 8 - \frac{8n(n+1)(2n+1)}{6n^3} \\ &= 8 - \frac{4}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) \\ \frac{1}{2}\text{Area} &= \lim_{n\to\infty} S(n) = 8 - \frac{8}{3} = \frac{16}{3} \\ \text{Area} &= \frac{32}{3} \end{aligned}$$