1025 HW-B

1. Find the points of inflection and discuss the concavity of the graph of the function

$$f(x) = x\sqrt{9-x}$$

2. Find the limit

$$\lim_{x\to\infty} x\tan\frac{1}{x}$$

3. Analyze and sketch a graph of the function (Label any intercepts,relative extrema, points of inflection,and asymptotes)

$$y = \frac{x^2+1}{x^2-4}$$

sol:

1.

$$\begin{aligned} f(x) &= x\sqrt{9-x}\ \text{, Domain:} x \le 9 \\ f'(x) &= \frac{3(6-x)}{2\sqrt{9-x}} \\ f''(x) &= \frac{3(x-12)}{4(9-x)^{\frac{3}{2}}} = 0 \text{when} x = 12 \end{aligned}$$

$x = 12$ is not in the domain. f'' is not contiouous at $x = 9$.

Interval:	$-\infty < x < 9$
Sign of $f''(x)$:	$f'' < 0$
Conclusion:	Concave downward

Concave downward: $(-\infty, 9)$

No point of inflection.

2.

$$\lim_{x\to\infty} x\tan\frac{1}{x} = \lim_{x\to 0^+}\frac{\tan t}{t} = \lim_{x\to 0^+}\left[\frac{\sin t}{t}\cdot\frac{1}{\cos t}\right] = (1)(1) = 1$$

3.

$$\begin{aligned} y &= \frac{x^2+1}{x^2-4} \\ y' &= \frac{-10x}{(x^2-4)^2} = 0 \text{ when } x = 0 \text{ and undefined when } x = \pm 2 \\ y'' &= \frac{10(3x^2+4)}{(x^2-4)^3} < 0 \text{ when } x = 0 \end{aligned}$$

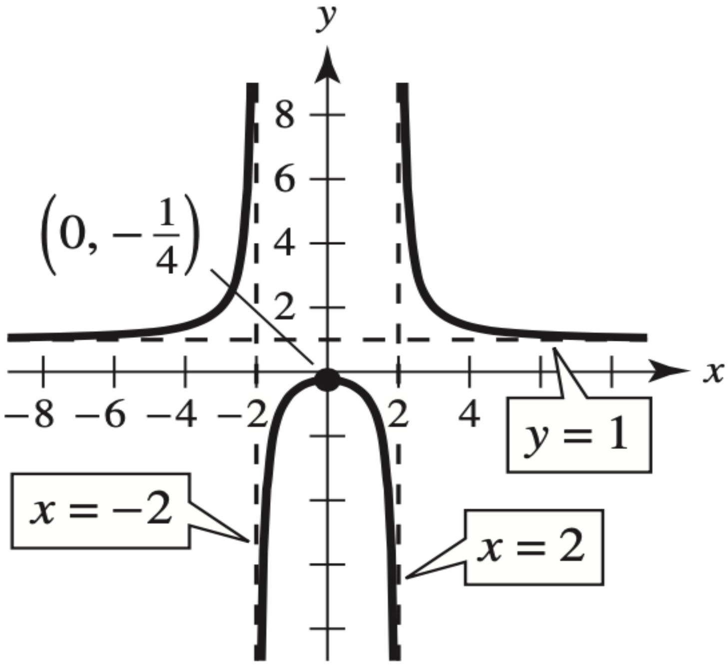
Intercept: $\left(0, -\frac{1}{4}\right)$

Symmetric about y-axis

Horizontal asymptote: $y = 1$

Vertical asymptotes: $x = \pm 2$

	y	y'	y''	Conclusion
$-\infty < x < -2$		$+$	$+$	Increasing,concave up
$-2 < x < 0$		$+$	$-$	Increasing,concave down
$x = 0$	$-\frac{1}{4}$			Relative maximum
$0 < x < 2$		$-$	$-$	Decreasing,concave down
$2 < x < \infty$		$-$	$+$	Decreasing,concave up


y
8
6
4
2
$\left(0, -\frac{1}{4}\right)$
x
−8
−6
−4
−2
2
4
$y = 1$
$x = -2$
$x = 2$