1025 HW-C

1. Find the points of inflection and discuss the concavity of the graph of the function

$$f(x) = x\sqrt{x+3}$$

2. Find the limit

$$\lim_{x\to\infty}(x - \sqrt{x^2+x})$$

3. Analyze and sketch a graph of the function (Label any intercepts,relative extrema, points of inflection,and asymptotes)

$$y = \frac{-x^2-4x-7}{x+3}$$

sol:

1.

$$\begin{aligned} f(x) &= x\sqrt{x+3}\ \text{, Domain:}[-3,\infty) \\ f'(x) &= x\left(\frac{1}{2}\right)(x+3)^{-\frac{1}{2}} + \sqrt{x+3} = \frac{3(x+2)}{2\sqrt{x+3}} \\ f''(x) &= \frac{6\sqrt{x+3} - 3(x+2)(x+3)^{-\frac{1}{2}}}{4(x+3)} \\ &= \frac{3(x+4)}{4(x+3)^{\frac{3}{2}}} = 0 \text{ when } x = -4 \end{aligned}$$

$x = -4$ is not in the domain. f'' is not contiouous at $x = -3$.

Interval:	$-3 < x < \infty$
Sign of $f''(x)$:	$f'' > 0$
Conclusion:	Concave upward

Concave upward: $(-3,\infty)$

There are no points of inflection.

2.

$$\begin{aligned} \lim_{x\to\infty}(x-\sqrt{x^2+x}) &= \lim_{x\to\infty}\left[(x-\sqrt{x^2+x})\cdot\frac{(x+\sqrt{x^2+x}}{(x+\sqrt{x^2+x}}\right] \\ &= \lim_{x\to\infty}\frac{-x}{x+\sqrt{x^2+x}} = \lim_{x\to\infty}\frac{-1}{1+\sqrt{1+(\frac{1}{x})}} \\ &= -\frac{1}{2} \end{aligned}$$

3.

$$\begin{aligned} y &= \frac{-x^2-4x-7}{x+3} = -x-1-\frac{4}{x+3} \\ y' &= -\frac{x^2+6x+5}{(x+3)^2} = -\frac{(x+1)(x+5)}{(x+3)^2} = 0 \\ &\text{when } x = -1,-5 \text{ and is undefined when } x = -3 \\ y'' &= \frac{-8}{(x+3)^3} < 0 \end{aligned}$$

Intercept: $\left(0,-\frac{7}{3}\right)$

No symmetry

Slant asymptote: $y = -x-1$

Vertical asymptotes: $x = -3$

	y	y'	y''	Conclusion
$-\infty < x < -5$		$-$	$+$	Decreasing,concave up
$x = -5$	6	0	$+$	Relative minimum
$-5 < x < -3$		$+$	$+$	Increasing,concave up
$-3 < x < -1$		$+$	$-$	Increasing,concave down
$x = -1$	-2	0	$-$	Relative maximum
$-1 < x < \infty$		$-$	$-$	Decreasing,concave down