1025 HW-A

1. Find the points of inflection and discuss the concavity of the graph of the function

$$f(x) = \frac{6-x}{\sqrt{x}}$$

2. Find the limit

$$\lim_{x\to\infty} \frac{5x^2+2}{\sqrt{x^2+3}}$$

3. Analyze and sketch a graph of the function (Label any intercepts,relative extrema, points of inflection,and asymptotes)

$$y = \frac{2x}{9-x^2}$$

sol:

1.

$$\begin{aligned} f(x) &= \frac{6-x}{\sqrt{x}} = 6x^{-\frac{1}{2}} - x^{\frac{1}{2}} \\ f'(x) &= \frac{-(x+6)}{2x^{\frac{3}{2}}} \\ f''(x) &= \frac{x+18}{4x^{\frac{5}{2}}} \end{aligned}$$

Note that the domain of f is $x > 0$.
Furthermore,$f''(x) > 0$ on $(0, \infty)$.
Concave upward: $(0, \infty)$
No points of inflection.

2.

$$\lim_{x\to\infty} \frac{5x^2+2}{\sqrt{x^2+3}} = \lim_{x\to\infty} \frac{5x^2+2}{x\sqrt{1+\left(\frac{3}{x^2}\right)}} = \lim_{x\to\infty} \frac{5x^2+\left(\frac{2}{x}\right)}{\sqrt{1+\left(\frac{3}{x^2}\right)}} = \infty$$

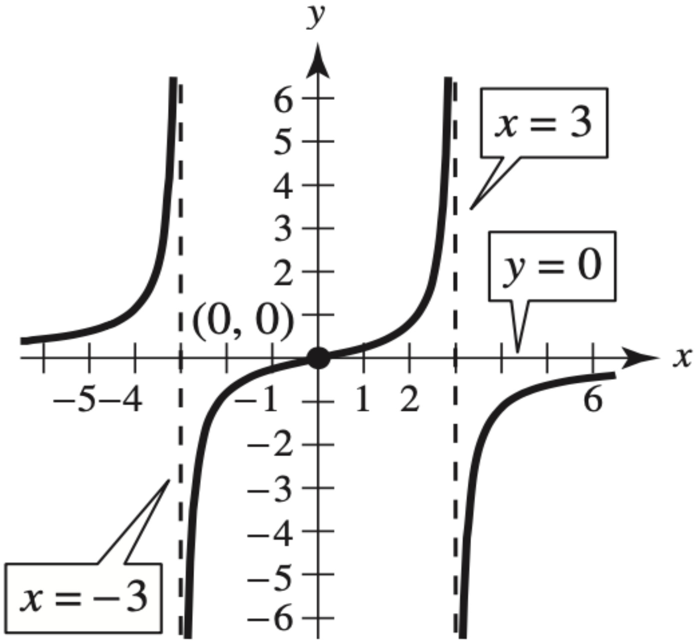
Limit does not exist.

3.

$$\begin{aligned} y &= \frac{2x}{9-x^2} \\ y' &= \frac{2(x^2+9)}{(x^2-9)^2}, \text{ undefined when } x = \pm 3 \\ y'' &= \frac{4x(x^2+27)}{(x^2-9)^3} = 0 \text{ when } x = 0, \text{undefined when } x = \pm 3 \end{aligned}$$

Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = \pm 3$

	y	y'	y''	Conclusion
$-\infty < x < -3$		$+$	$+$	Increasing,concave up
$-3 < x < 0$		$+$	$-$	Increasing,concave down
$x = 0$	0	$+$	0	Point of inflection
$0 < x < 3$		$+$	$+$	Increasing,concave up
$3 < x < \infty$		$+$	$-$	Increasing,concave down


y
x
6
5
4
3
2
(0, 0)
−5−4
−1
1 2
6
−2
−3
−4
−5
−6
x = 3
y = 0
x = −3