1018 HW-B

1. Find the derivative
$$f(x) = 5\cos^2(\pi x)$$

2. Find the $\dfrac{dy}{dx}$ by implicit differentiation
$$x^3y^3 - y = x$$

3. Find the absolute extrema of the function on the closed interval
$$f(x) = 3x^{\frac{2}{3}} - 2x, [-1, 1]$$

4.
$$f(x) = \frac{x^2 - 2x + 1}{x + 1}$$
   (a) Find the critical numbers of f
   (b) Find the open intervals on which the function is increasing or decreasing
   (c) Apply the First Derivative Test to identify all relative extrema

sol:

1.
$$\begin{aligned} f(x) &= 5\cos^2(\pi x) = 5(\cos \pi x)^2 \\ f'(x) &= 10\cos \pi x(-\sin \pi x)(\pi) \\ &= -10\pi(\sin \pi x)(\cos \pi x) \\ &= -5\pi \sin 2\pi x \end{aligned}$$

2.
$$\begin{aligned} x^3y^3 - y - x &= 0 \\ 3x^3y^2y' + 3x^2y^3 - y' - 1 &= 0 \\ (3x^3y^2 - 1)y' &= 1 - 3x^2y^3 \\ y' &= \frac{1 - 3x^2y^3}{3x^3y^2 - 1} \end{aligned}$$

3.
$$\begin{aligned} f(x) &= 3x^{\frac{2}{3}} - 2x, [-1, 1] \\ f'(x) &= 2x^{-\frac{1}{3}} - 2 = \frac{2\,(1 - \sqrt[3]{x})}{\sqrt[3]{x}} \end{aligned}$$

Left endpoint: $(-1, 5)$ Maximum
Critical number:$(0, 0)$ Minimum
Right endpoint: $(1, 1)$

4. (a)
$$\begin{aligned} f(x) &= \frac{x^2 - 2x + 1}{x + 1} \\ f'(x) &= \frac{(x+1)(2x-2) - (x^2 - 2x + 1)(1)}{(x+1)^2} = \frac{x^2 + 2x - 3}{(x+1)^2} = \frac{(x+3)(x-1)}{(x+1)^2} \end{aligned}$$

Critical number: $x = -3, 1$
Discontinuity: $x = -1$

   (b)

Test intervals:	$-\infty < x < -3$	$-3 < x < -1$	$-1 < x < 1$	$1 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Decreasing	Increasing

Increasing on: $(-\infty, -3), (1, \infty)$

Decreasing on: $(-3, -1), (-1, 1)$

(c)

Relative maximum: $(-3, -8)$

Relative minimum: $(1, 0)$