1018 HW-A

1. Find the derivative

$$f(x) = \sec(\frac{x}{2})\tan(\frac{x}{2})$$

2. Find the $\frac{dy}{dx}$ by implicit differentiation

$$x^4y - 8xy + 3xy^2 = 9$$

3. Find the absolute extrema of the function on the closed interval

$$f(x) = \frac{x}{x+3}, [-1, 6]$$

4.

$$f(x) = 5 - |x - 5|$$

(a) Find the critical numbers of f
(b) Find the open intervals on which the function is increasing or decreasing
(c) Apply the First Derivative Test to identify all relative extrema

sol:

1.

$$\begin{aligned} f'(x) &= \sec(\frac{x}{2})\sec^2(\frac{x}{2})\frac{1}{2} + \tan(\frac{x}{2})\sec(\frac{x}{2})\tan(\frac{x}{2})\frac{1}{2} \\ &= \frac{1}{2}\sec(\frac{x}{2})\left[\sec^2(\frac{x}{2}) + \tan^2(\frac{x}{2})\right] \end{aligned}$$

2.

$$\begin{aligned} x^4y - 8xy + 3xy^2 &= 9 \\ x^4y' + 4x^3y - 8xy' - 8y + 6xyy' + 3y^2 &= 0 \\ (x^4 - 8x + 6xy)y' &= 8y - 4x^3y - 3y^2 \\ y' &= \frac{8y - 4x^3y - 3y^2}{x^4 - 8x + 6xy} \end{aligned}$$

3.

$$\begin{aligned} f(x) &= \frac{x}{x+3}, [-1, 6] \\ f'(x) &= \frac{(x+3)(1) - x(1)}{(x+3)^2} = \frac{3}{(x+3)^2} \end{aligned}$$

No critical numbers

Left endpoint: $\left(-1, -\frac{1}{2}\right)$ Minimum

Right endpoint: $\left(6, \frac{2}{3}\right)$ Maximum

4. (a)

$$\begin{aligned} f(x) &= 5 - |x - 5| \\ f'(x) &= -\frac{x-5}{|x-5|} = \begin{cases} 1, & x < 5 \\ -1, & x > 5 \end{cases} \end{aligned}$$

Critical number: $x = 5$

(b)

Test intervals:	$-\infty < x < 5$	$5 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 5)$
Decreasing on: $(5, \infty)$

(c)

Relative maximum: $(5, 5)$