1018 HW-C

1. Find the derivative
$$f(x) = \frac{1}{4}\sin^2(2x)$$

2. Find the $\frac{dy}{dx}$ by implicit differentiation
$$x^2y + y^2x = -2$$

3. Find the absolute extrema of the function on the closed interval
$$f(x) = \sqrt[3]{x}, [-8, 8]$$

4.
$$f(x) = \begin{cases} 4 - x^2, & x \leq 0 \\ -2x, & x > 0 \end{cases}$$

   (a) Find the critical numbers of f
   (b) Find the open intervals on which the function is increasing or decreasing
   (c) Apply the First Derivative Test to identify all relative extrema

sol:

1.
$$\begin{aligned} f(x) &= \frac{1}{4}\sin^2(2x) = \frac{1}{4}(\sin 2x)^2 \\ f'(x) &= 2(\frac{1}{4})(\sin 2x)(\cos 2x)(2) \\ &= \sin 2x \cos 2x \\ &= \frac{1}{2}\sin 4x \end{aligned}$$

2.
$$\begin{aligned} x^2y + y^2x &= -2 \\ x^2y' + 2xy + y^2 + 2yxy' &= 0 \\ (x^2 + 2xy)y' &= -(y^2 + 2xy) \\ y' &= \frac{-y(y + 2x)}{x(x + 2y)} \end{aligned}$$

3.
$$\begin{aligned} f(x) &= \sqrt[3]{x}, [-8, 8] \\ f'(x) &= \frac{1}{3x^{\frac{2}{3}}} \end{aligned}$$

Left endpoint: $(-8, -2)$ Minimum
Critical number:$(0, 0)$
Right endpoint: $(8, 2)$ Maximum

4. (a)
$$f(x) = \begin{cases} 4 - x^2, & x \leq 0 \\ -2x, & x > 0 \end{cases}$$
$$f'(x) = \begin{cases} -2x, & x < 0 \\ -2, & x > 0 \end{cases}$$

Critical number: $x = 0$

   (b)

Test intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 0)$

Decreasing on: $(0, \infty)$

(c)

Relative maximum: $(0, 4)$