

1004 HW-C

1. Find the vertical asymptotes

$$f(x) = \frac{3}{x^2 + x - 2}$$

2. Find the derivative by the limit process

$$f(x) = \sqrt{x+4}$$

3. Find the derivative

(a)

$$f(x) = \frac{3x^2 + 4x - 8}{\sqrt{x^3}}$$

(b)

$$f(x) = \frac{1}{x} - 12 \sec x$$

sol:

- 1.

$$f(x) = \frac{3}{x^2 + x - 2} = \frac{3}{(x+2)(x-1)}$$

$$\lim_{x \rightarrow -2^-} \frac{3}{x^2 + x - 2} = \infty \text{ and } \lim_{x \rightarrow -2^+} \frac{3}{x^2 + x - 2} = -\infty$$

Therefore, $x = -2$ is a vertical asymptote.

$$\lim_{x \rightarrow 1^-} \frac{3}{x^2 + x - 2} = -\infty \text{ and } \lim_{x \rightarrow 1^+} \frac{3}{x^2 + x - 2} = \infty$$

Therefore, $x = 1$ is a vertical asymptote.

- 2.

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x + 4} - \sqrt{x + 4}}{\Delta x} \cdot \left(\frac{\sqrt{x + \Delta x + 4} + \sqrt{x + 4}}{\sqrt{x + \Delta x + 4} + \sqrt{x + 4}} \right) \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x + 4) - (x + 4)}{\Delta x [\sqrt{x + \Delta x + 4} + \sqrt{x + 4}]} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x + 4} + \sqrt{x + 4}} \\ &= \frac{1}{\sqrt{x + 4} + \sqrt{x + 4}} \\ &= \frac{1}{2\sqrt{x + 4}} \end{aligned}$$

3. (a)

$$\begin{aligned}f(x) &= \frac{3x^2 + 4x - 8}{\sqrt{x^3}} = 3\sqrt{x} + \frac{4}{\sqrt{x}} - \frac{8}{\sqrt{x^3}} \\f'(x) &= \frac{3}{2\sqrt{x}} - \frac{2}{\sqrt{x^3}} + \frac{12}{\sqrt{x^5}} \\&= \frac{3x^2 - 4x + 24}{2\sqrt{x^5}}\end{aligned}$$

(b)

$$\begin{aligned}f(x) &= \frac{1}{x} - 12 \sec x = x^{-1} - 12 \sec x \\f'(x) &= -x^{-2} - 12 \sec x \tan x \\&= \frac{-1}{x^2} - 12 \sec x \tan x\end{aligned}$$