1004 HW-B

1. Find the vertical asymptotes

$$f(x) = \frac{x^2 - 5x + 25}{x^3 + 125}$$

2. Find the derivative by the limit process

$$f(x) = x^3 - 12x$$

3. Find the derivative

   (a)
   $$f(x) = x^2 - \frac{4}{x^3}$$

   (b)
   $$f(x) = \frac{\sec x}{x}$$

sol:

1.
$$f(x) = \frac{x^2 - 5x + 25}{x^3 + 125} = \frac{x^2 - 5x + 25}{(x + 5)(x^2 - 5x + 25)} = \frac{1}{x + 5}$$

$$\lim_{x \to -5^-} \frac{1}{x + 5} = -\infty \text{ and } \lim_{x \to -5^+} \frac{1}{x + 5} = \infty$$

Therefore, $x = -5$ is a vertical asymptote.

2.
$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 - 12(x + \Delta x)\right] - \left[x^3 - 12x\right]}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12x - 12\Delta x - x^3 + 12x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \left(3x^2 + 3x\Delta x + (\Delta x)^2 - 12\right) \\
&= 3x^2 - 12
\end{aligned}$$

3. (a)
$$f(x) = x^2 - \frac{4}{x^3} = x^2 - 4x^{-3}$$
$$f'(x) = 2x + 12x^{-4} = 2x + \frac{12}{x^4}$$

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(b)

$$f(x) = \frac{\sec x}{x}$$
$$f'(x) = \frac{x \sec x \tan x - \sec x}{x^2}$$
$$= \frac{\sec x (x \tan x - 1)}{x^2}$$