

1004 HW-A

1. Find the vertical asymptotes

$$f(x) = \frac{3x+4}{x^2-16}$$

2. Find the derivative by the limit process

$$f(x) = \frac{1}{x-1}$$

3. Find the derivative

(a)

$$f(x) = \frac{x^3 - 3x^2 + 4}{x^2}$$

(b)

$$f(x) = \frac{3(1 - \sin x)}{2 \cos x}$$

sol:

- 1.

$$f(x) = \frac{3x+4}{x^2-16} = \frac{3x+4}{(x-4)(x+4)}$$

$$\lim_{x \rightarrow 4^-} \frac{3x+4}{x^2-16} = -\infty \text{ and } \lim_{x \rightarrow 4^+} \frac{3x+4}{x^2-16} = \infty$$

Therefore, $x = 4$ is a vertical asymptote.

$$\lim_{x \rightarrow -4^-} \frac{3x+4}{x^2-16} = -\infty \text{ and } \lim_{x \rightarrow -4^+} \frac{3x+4}{x^2-16} = \infty$$

Therefore, $x = -4$ is a vertical asymptote.

- 2.

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{x + \Delta x - 1} - \frac{1}{x - 1}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x - 1) - (x + \Delta x - 1)}{\Delta x(x + \Delta x - 1)(x - 1)} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\Delta x}{\Delta x(x + \Delta x - 1)(x - 1)} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-1}{(x + \Delta x - 1)(x - 1)} \\ &= -\frac{1}{(x - 1)^2} \end{aligned}$$

3. (a)

$$f(x) = \frac{x^3 - 3x^2 + 4}{x^2} = x - 3 + 4x^{-2}$$

$$f'(x) = 1 - \frac{8}{x^3} = \frac{x^3 - 8}{x^3}$$

(b)

$$f(x) = \frac{3(1 - \sin x)}{2 \cos x} = \frac{3 - 3 \sin x}{2 \cos x}$$

$$f'(x) = \frac{(-3 \cos x)(2 \cos x) - (3 - 3 \sin x)(-2 \sin x)}{(2 \cos x)^2}$$

$$= \frac{-6 \cos^2 x + 6 \sin x - 6 \sin^2 x}{4 \cos^2 x}$$

$$= \frac{3}{2}(-1 + \tan x \sec x - \tan^2 x)$$

$$= \frac{3}{2} \sec x (\tan x - \sec x)$$