

Find or evaluate the integral

1. $\int \frac{x+1}{\sqrt{3x^2+6x}} dx$

2. $\int_2^3 \frac{\ln(x+1)^3}{x+1} dx$

3. $\int \frac{xe^{2x}}{(2x+1)^2} dx$

4. $\int \sin \sqrt{x} dx$

sol:

1.

Let $u = 3x^2 + 6x, du = (6x + 6)dx = 6(x + 1)dx$

$$\begin{aligned} \int \frac{x+1}{\sqrt{3x^2+6x}} dx &= \frac{1}{6} \int (3x^2 + 6x)^{-1/2} 6(x+1) dx \\ &= \frac{1}{6} \cdot \frac{\sqrt{3x^2+6x}}{1/2} + C \\ &= \frac{1}{3} \sqrt{3x^2+6x} + C \end{aligned}$$

2.

$$\begin{aligned} \int_2^3 \frac{\ln(x+1)^3}{x+1} dx &= 3 \int_2^3 \ln(x+1) \frac{1}{x+1} dx \\ &= \frac{3[\ln(x+1)]^2}{2} \Big|_2^3 \\ &= \frac{3}{2}[(\ln 4)^2 - (\ln 3)^2] \end{aligned}$$

3.

$$\begin{aligned} dv &= \frac{1}{(2x+1)^2} dx \Rightarrow v = -\frac{1}{2(2x+1)} \\ u &= xe^{2x} \Rightarrow du = e^{2x}(2x+1)dx \\ \int \frac{xe^{2x}}{(2x+1)^2} dx &= -\frac{xe^{2x}}{2(2x+1)} + \int \frac{e^{2x}}{2} dx \\ &= \frac{-xe^{2x}}{2(2x+1)} + \frac{e^{2x}}{4} + C \\ &= \frac{e^{2x}}{4(2x+1)} + C \end{aligned}$$

4.

$$\begin{aligned} u &= \sqrt{x} \Rightarrow u^2 = x \Rightarrow 2u du = dx \\ \int \sin \sqrt{x} dx &= 2 \int u \sin u du \\ w &= u, dw = du, dv = \sin u du, v = -\cos u \\ 2 \int u \sin u du &= 2(-u \cos u + \int \cos u du) \\ &= 2(-u \cos u + \sin u) + C \\ &= 2(-\sqrt{x} \cos \sqrt{x} + \sin \sqrt{x}) + C \end{aligned}$$