

Find or evaluate the integral

1. $\int \frac{4}{3 - e^x} dx$

2. $\int_1^3 \frac{2x^2 + 3x - 2}{x} dx$

3. $\int 4 \arccos x dx$

4. $\int 2x^3 \cos x^2 dx$

sol:

1.

$$\begin{aligned} \int \frac{4}{3 - e^x} dx &= 4 \int \frac{e^{-x}}{3e^{-x} - 1} dx \\ &= -\frac{4}{3} \ln |3e^{-x} - 1| + C \end{aligned}$$

2.

$$\begin{aligned} \int_1^3 \frac{2x^2 + 3x - 2}{x} dx &= \int_1^3 \left(2x + 3 - \frac{2}{x} \right) dx \\ &= [x^2 + 3x - 2 \ln |x|]_1^3 \\ &= (9 + 9 - 2 \ln 3) - (1 + 3 - 0) \\ &= 14 - 2 \ln 3 \end{aligned}$$

3.

$$\begin{aligned} dv &= dx \Rightarrow v = x \\ u &= \arccos x \Rightarrow du = -\frac{1}{\sqrt{1 - x^2}} dx \\ 4 \int \arccos x dx &= 4 \left(x \arccos x + \int \frac{x}{\sqrt{1 - x^2}} dx \right) \\ &= 4 \left(x \arccos x - \sqrt{1 - x^2} \right) + C \end{aligned}$$

4.

$$\begin{aligned} u &= x^2, du = 2x dx \\ \int 2x^3 \cos x^2 dx &= \int x^2 \cos(x^2)(2x)dx = \int u \cos u du \\ w &= u, dw = du, dv = \cos u du, v = \sin u \\ \int u \cos u du &= u \sin u - \int \sin u du \\ &= u \sin u + \cos u + C \\ &= x^2 \sin x^2 + \cos x^2 + C \end{aligned}$$