1220 HW-C

Find or evaluate the integral

1. $\int \dfrac{1}{25 + 4x^2}\, dx$

2. $\int_1^e \dfrac{1 - \ln x}{x}\, dx$

3. $\int \dfrac{5x}{e^{2x}}\, dx$

4. $\int e^{\sqrt{2x}}\, dx$

sol:

1.

$$\text{Let } u = 2x, du = 2dx, a = 5$$

$$\int \dfrac{1}{25 + 4x^2}\, dx = \dfrac{1}{2} \int \dfrac{1}{5^2 + (2x)^2}(2)\, dx$$

$$= \dfrac{1}{10} \arctan \dfrac{2x}{5} + C$$

2.

$$\text{Let } u = 1 - \ln x, du = \dfrac{-1}{x} dx$$

$$\int_1^e \dfrac{1 - \ln x}{x}\, dx = -\int_1^e (1 - \ln x)\left(\dfrac{-1}{x}\right)\, dx$$

$$= \left[ -\dfrac{1}{2}(1 - \ln x)^2 \right]_1^e$$

$$= \dfrac{1}{2}$$

3.

$$dv = e^{-2x} dx \Rightarrow v = \int e^{-2x} dx = -\dfrac{1}{2}e^{-2x}$$

$$u = 5x \Rightarrow du = 5dx$$

$$\int e^{\sqrt{2x}}\, dx = \int 5xe^{-2x}\, dx$$

$$= (5x)\left(-\dfrac{1}{2}e^{-2x}\right) - \int \left(-\dfrac{1}{2}e^{-2x}\right) 5\, dx$$

$$= -\dfrac{5}{2}e^{-2x} + \dfrac{5}{2}\int e^{-2x}\, dx$$

$$= -\dfrac{5}{2}e^{-2x} - \dfrac{5}{4}e^{-2x} + C$$

$$= -\dfrac{5}{4}e^{-2x}(2x + 1) + C$$

4.

$$\text{Let } u = \sqrt{2x}, u^2 = 2x, 2u\, du = 2dx$$

$$\int e^{\sqrt{2x}}\, dx = \int e^u (u\, du)$$

$$dv = e^u\, du \Rightarrow v = \int e^u\, du = e^u$$

$$w = u \Rightarrow de = du$$

$$\int e^{\sqrt{2x}}\, dx = ue^u - \int e^u\, du$$

$$= ue^u - e^u + C$$

$$= \sqrt{2x}e^{\sqrt{2x}} - e^{\sqrt{2x}} + C$$